

## DEPENDENCE OF THE CONTACT HEAT CONDUCTIVITY OF GRANULAR SYSTEMS ON THE EXTERNAL LOAD

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An examination has been made of the dependence of the contact heat conductivity of granular systems on the external load. The calculation formulas proposed for contact heat conduction are applicable over a wide range of materials.

Results have been reported of a number of investigations of contact heat transfer between flat machined surfaces [1], whereas there is practically no information on the contact heat conduction of granular systems.

In a granular system contacts are formed through micro-roughnesses of the surface of the particles in contact. The actual area of such a contact,  $s_a$ , is the sum of the individual areas. The contact thermal conductivity  $\sigma_c$  is defined by the relation

$$\sigma_c = \lambda_s s_a / h_r \quad (1)$$

We will suppose that in the arrangement of the particles of the granular system long-range order is observed, and the particles form the densest packing (tetragonal). It has been shown in [2] that the effective thermal conductivity of such a system is equal to the effective thermal conductivity of the elementary cell. In the system examined this is a rectangular parallelepiped, having a base square of side  $d = 2r$ , and a height  $H = d(2)^{1/2}$ . The value of the contact thermal conductivity in the elementary cell may be represented by the relation [2]

$$\lambda_c = \frac{1.41}{rA} \sigma_c, \quad A = \sqrt[3]{\frac{75}{100-p}} \quad (2)$$

From (1) and (2) it follows that

$$\lambda_c = \frac{1.41}{rA} \frac{s_a}{h_r k_1} \quad (3)$$

where  $k_1$  is a coefficient depending on the relative position of the micro-roughnesses in contact. It may be shown that the value of this coefficient  $1 \leq k_1 \leq 2$  (Fig. 1).

We divide the area of actual contact  $s_a$  into an area  $s_{a1}$ , corresponding to the freely poured state, and an

additional area  $s_{a2}$  arising from the action of an external load. Then we write (3) as follows:

$$\lambda_c = \frac{1.41}{rA} \lambda_s \frac{s_{a1} + s_{a2}}{h_r k_1} \quad (4)$$

We represent the contact thermal conductivity of granular materials as the sum of two components—a constant one and a variable one:

$$\lambda_c = \lambda_f + \lambda(\Delta) \quad (5)$$

The quantity  $\lambda_f$  is determined experimentally from tests with granular systems under conditions of high vacuum and absence of an external load. On the basis of (4), the second term of (5) has been represented in the form

$$\lambda(\Delta) = \frac{1.41}{rA} \lambda_s \frac{s_{a2}}{h_r k_1} \quad (6)$$

In (6) the parameters  $s_{a2}$ ,  $h_r$  and  $k_1$ , which depend in turn on the value of the load, are unknown. We will examine the structure of these parameters.

As has already been noted, the contact between solid bodies is always discrete in nature, the area of the actual contact being a small fraction ( $10^{-2}$ – $10^{-5}$ ) of the nominal area of contact,  $s_n$ , and proportional to it [3–5]:

$$s_a = \eta s_n \quad (7)$$

Using the Hertz formula [6] for the chosen packing of the spheres, we find the value of the nominal area of contact of two ideal elastic spheres under the influence of a load  $\Delta$ :

$$s_n = 0.176 \pi d^2 (\Delta/E)^{2/3} \quad (8)$$

The nominal area of contact of actual spheres will be larger, since the effective modulus of elasticity of the surface layer (composed of micro-roughnesses and the gas spaces between them) is less than that of the material of the spheres. We will consider that the effective modulus of elasticity of the surface layer is equal to that of the granular system  $E_0$ .

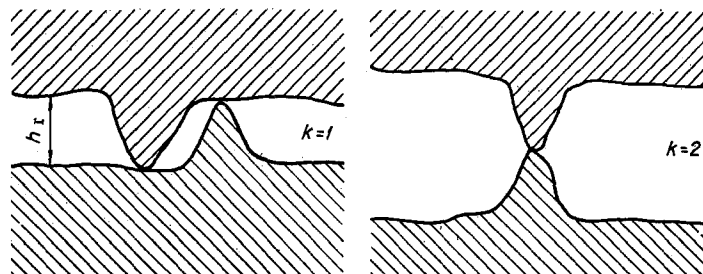


Fig. 1. Relative position of roughnesses in contact.

Table 1

Physical and Mechanical Characteristics of the Materials Examined

Material:	d, mm	h <sub>r</sub> , mm	2h <sub>r</sub> /d	k <sub>3</sub>
Gravel	5-7	(6-7) · 10 <sup>-3</sup>	2 · 10 <sup>-3</sup>	2
Glass beads	2.5	(8-10) · 10 <sup>-3</sup>	(6-8) · 10 <sup>-3</sup>	6-8
ZnSO <sub>4</sub>	3-4	(4-7) · 10 <sup>-3</sup>	(2-4) · 10 <sup>-3</sup>	2-4
Lead shot	3.5	(2-5) · 10 <sup>-3</sup>	(2-4) · 10 <sup>-3</sup>	2-4

Experiments performed indicate that for various materials (glass and steel balls, lead shot, and quartz sand), at specific loads up to 2 · 10<sup>5</sup> N/m<sup>2</sup>, the value of E<sub>0</sub> is constant and equal to 650 · 10<sup>5</sup> N/m<sup>2</sup>.

According to the data of the experiments, at large values of the specific load (from 3 · 10<sup>5</sup> to 16 · 10<sup>5</sup> N/m<sup>2</sup>), the modulus of elasticity of the granular system E<sub>0</sub> varies as follows with increase in specific load:

$$E_0 = k_2 \Delta^{1/3} E^{2/3}, \quad 3 \leq \Delta \cdot 10^{-5} \leq 16 \text{ N/m}^2, \quad (9)$$

where k<sub>2</sub> is an empirical coefficient, varying in the range 0.5 to 1.0 for the different materials.

Replacing E in (8) by E<sub>0</sub>, we obtain the value of the nominal area of contact under loads from 0 to 3 · 10<sup>5</sup> N/m<sup>2</sup>

$$s_n = 0.93 \cdot 10^{-2} \pi r^2 \Delta^{2/3}, \quad (10)$$

where the numerical coefficient has dimensions [N<sup>-2/3</sup> · m<sup>4/3</sup>]. Under loads from 3 · 10<sup>5</sup> to 16 · 10<sup>5</sup> N/m<sup>2</sup>, the value of the nominal area of contact is determined from the formula

$$s_n = 0.56 \pi r^2 (\Delta/E)^{1/3} \cdot 1/k_2^{2/3}. \quad (11)$$

The area of actual contact depends on the specific load, the hardness characteristics of the bodies in contact, and the micro-geometry of the surface contacts, i. e., is determined by diverse factors which are difficult to calculate simultaneously. We will therefore find the area of contact η experimentally, together with the other experimental coefficients k<sub>1</sub> and k<sub>2</sub>, which will be discussed below.

A second unknown parameter in (6) is the height of a micro-roughness, h<sub>r</sub>. For unfinished surfaces it may be defined tentatively. If the material of which the granular system is composed has a machined surface, then h<sub>r</sub> is determined according to the degree of finish of the surface. If the surface of the material has not been machined, then h<sub>r</sub> is determined from the test. Table 1 gives results of measurements of h<sub>r</sub> for various granular materials.

It may be seen from the table that the ratio h<sub>r</sub>/r is very stable for the different materials and may be represented in the following form:

$$h_r/r = k_3 \cdot 10^{-3}, \quad (12)$$

where k<sub>3</sub> is an empirical coefficient (see Table 1).

Substituting the values of h<sub>r</sub> and s<sub>a2</sub> into (6), we obtain the dependence of the contact thermal conductivity on the external load

$$\lambda(\Delta) = \lambda_s \frac{1}{75A} \Delta^{1/3} k_m, \quad (13)$$

where

$$k_m = 3.1 \cdot 10^3 \eta/k_1 k_3, \quad \Delta = 0 - 3 \cdot 10^5 \text{ N/m}^2.$$

Under loads of (3-16) · 10<sup>5</sup> N/m<sup>2</sup>

$$\lambda(\Delta) = \frac{\lambda_s}{E^{1/3}} \frac{1}{A} \Delta^{1/3} k_b, \quad (14)$$

where k<sub>b</sub> = 3.1 · 10<sup>3</sup> η/k<sub>2</sub><sup>2/3</sup> k<sub>1</sub> k<sub>3</sub>.

Coefficients k<sub>m</sub> and k<sub>b</sub> are determined from experiment. An experimental investigation of the dependence of the contact thermal conductivity on the external load was conducted on an instrument specially created for this purpose. Its main part is a steady-state calorimeter. The pressure of the pore gas in the material under examination was 0.1 · 10<sup>-2</sup> - 1.0 · 10<sup>-2</sup> N/m<sup>2</sup>. The mean temperature of the test material was approximately 300° K. The value of the external load was varied in the range 0 to 16 · 10<sup>5</sup> N/m<sup>2</sup>. Measurement of the thermal conductivity was performed at a fixed load and under steady thermal conditions.

The parameters of the materials investigated are presented in Table 2.

As may be seen from the table, the chosen materials span a wide range of variation of λ<sub>s</sub> and of the group λ<sub>s</sub>/E<sup>1/3</sup>.

The values of λ obtained from experiment at the various loads were reduced in conformity with formulas (13) and (14), allowing us to determine values

Table 2

Experimental Values of Contact Thermal Conductivity of the Materials Examined with No External Load

Material	λ <sub>s</sub> , w/m degree	E · 10 <sup>-5</sup> , n/m <sup>2</sup>	(λ <sub>s</sub> /E <sup>1/3</sup> ) · 10 <sup>3</sup> , w degree <sup>-1</sup> · n <sup>-4/3</sup> · m <sup>1/3</sup>
Glass beads . . . . .	0.84	5 · 10 <sup>5</sup>	0.255 · 10 <sup>-2</sup>
Granite . . . . .	1.0	3.7 · 10 <sup>5</sup>	0.35 · 10 <sup>-2</sup>
Quartz sand . . . . .	4.0	4.3 · 10 <sup>5</sup>	1.4 · 10 <sup>-2</sup>
Steel balls . . . . .	40	20 · 10 <sup>5</sup>	6.5 · 10 <sup>-2</sup>

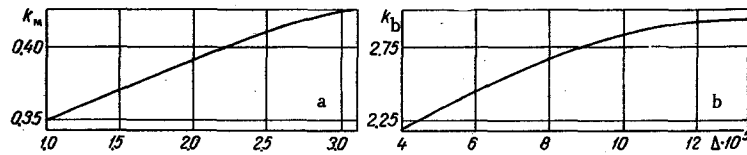


Fig. 2. Dependence of the mean values of coefficients  $k_m$  (a) and  $k_b$  (b) on the load  $\Delta$  ( $N/m^2$ ).

of coefficients  $k_m$  and  $k_b$ , and to establish that they are practically independent of the material of the grains and their size. The dependence obtained of the coefficients  $k_m$  and  $k_b$  on the external load (Fig. 2) permits us to extend the semi-empirical relations (13) and (14) to other materials. Since elastic contact of the particles was assumed in the derivation of these formulas, the dependences obtained should apply only to materials which undergo elastic deformation.

Thus, we may propose the following formulas for calculating the contact thermal conductivity of granular materials as a function of the external load: for loads from 0 to  $3 \cdot 10^5$   $N/m^2$

$$\lambda_c = \lambda_f + \frac{\lambda_s}{75} \frac{1}{A} \Delta^{2/3} k_m, \quad (15)$$

for loads from  $3 \cdot 10^5$  to  $16 \cdot 10^5$   $N/m^2$

$$\lambda_c = \lambda_f + \frac{\lambda_s}{E^{1/2}} \frac{1}{A} \Delta^{1/2} k_b. \quad (16)$$

The value of  $\lambda_f$  for particles of mineral origin ( $\lambda_s < 1-2$   $W/m \cdot \text{degree}$ ) is  $(2-10) \cdot 10^{-3}$ , and for a system of metal particles— $(3-8) \cdot 10^{-2}$   $W/m \cdot \text{degree}$ .

#### NOTATION

$s_a$  is the area of actual contact of two particles in a granular material;  $\sigma_c$  is the conductivity of the contact between two particles;  $h_r$  is the height of a micro-roughness;  $\lambda_s$  is the thermal conductivity of the material of the particles;  $d = 2r$  is the particle diameter;  $\lambda_c$  is the contact thermal conductivity of the

granular material;  $p$  is the porosity of the system;  $s_{a1}$ ,  $s_{a2}$  is the area of contact of two particles in the freely poured state and under the action of a load;  $\lambda_f$  is the thermal conductivity of the granular system in the freely poured state;  $\lambda(\Delta)$  is the portion of the thermal conductivity of a granular material that depends on the external load;  $\eta$  is the relative area of contact;  $s_n$  is the nominal area of contact of the two particles;  $\Delta$  is the external specific load;  $E$  is the modulus of elasticity of the particle material;  $E_0$  is the effective modulus of elasticity of the granular material;  $k_1$ ,  $k_2$ ,  $k_3$ ,  $k_m$ ,  $k_b$  are the empirical coefficients.

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3 January 1966

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